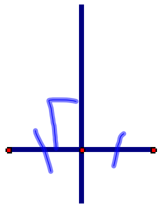
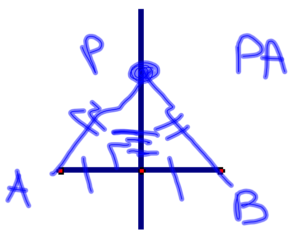
Ch 5 Relationships in Triangles
5.1 Bisectors, Medians, and Altitudes

perpendicular bisector-of a side of a triangle is a line segment or ray that passes through the midpoint of the side and is perpendicular to it.



Thm 5.1-Any point on the perpendicular bisector of a segment is equidistant from the endpoints of the segment.



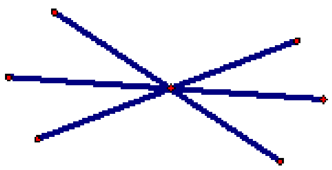

PA = PB

Thm 5.2-Any point equidistant from the endpoints of the segment lies on the perpendicular bisector of the segment.

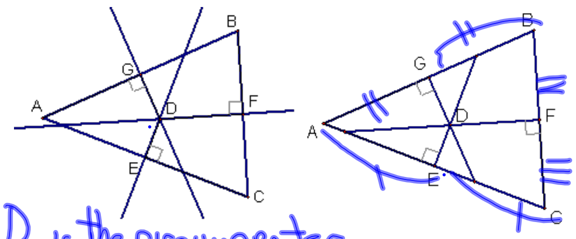
(converse 5.1)

Concurrent lines-three or more lines that intersect at a common point

Point of concurrency-the point of intersection



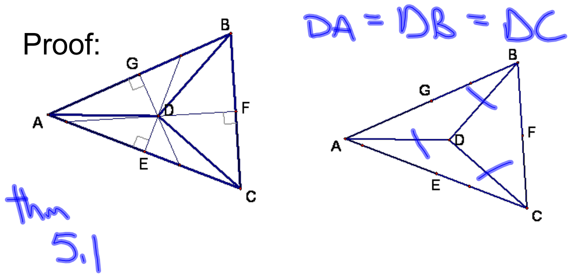
Circumcenter-the point of concurrency of the perpendicular bisectors of a triangle.




D is the circumcenter

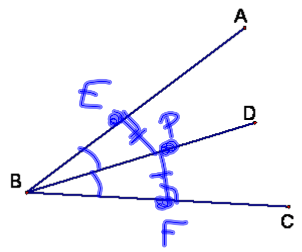
Thm 5.3 Circumcenter theorem-The circumcenter of a triangle is equidistant from the vertices of the triangle.

Proof:



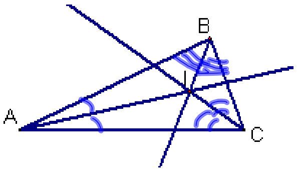

Thm 5.4-Any point on the angle bisector is equidistant from the sides of the angle.

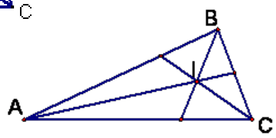
Thm 5.5-Any point equidistant from the sides of an angle lies on the angle bisector.



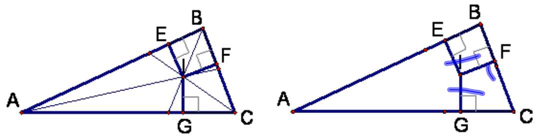

PE = PF

Incenter-The point of concurrency of the angle bisectors of a triangle.



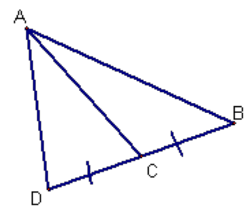




Thm 5.6 The Incenter Theorem-The incenter of a triangle is equidistant from each side of the triangle.



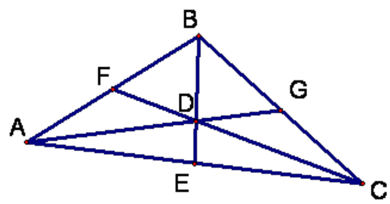

EI = FI = GI

Median-is a segment whose endpoint are the vertex of a triangle and the midpoint of the side opposite the vertex.




$\overline{AC}$ is a median

Centroid-The point of concurrency of the three medians of a triangle.



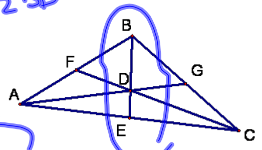

(point of balance)

Thm 5.7 The Centroid Theorem-The centroid of a triangle is located two-thirds of the distance from a vertex to the midpoint of the side opposite the vertex on a median.

$DE = \frac{1}{2} BD$

Conclusion:

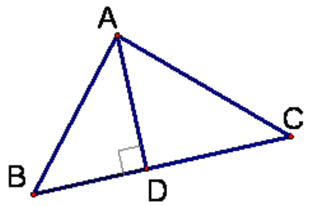
| | |
|---|---|
| DB = $\frac{2}{3}$ BE | DE = $\frac{1}{3}$ BE |
| DA = $\frac{2}{3}$ AG | DG = $\frac{1}{3}$ AG |
| CD = $\frac{2}{3}$ CF | FD = $\frac{1}{3}$ CF |



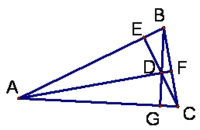

$DG = \frac{1}{2} AD$

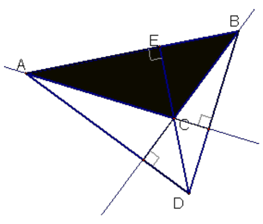
$FD = \frac{1}{2} CD$

Altitude-of a triangle is a segment form a vertex to the line containing the opposite side and perpendicular to the line containing that side.



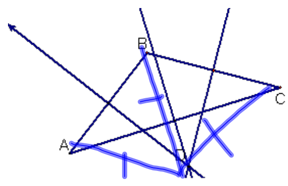

Orthocenter-The point of concurrency of the three altitudes of a triangle.

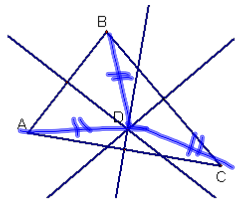



**Summary**
The circumcenter. (3 ⊥ bisectors)

$\overline{DA}$ ≅ $\overline{DB}$ ≅ $\overline{DC}$

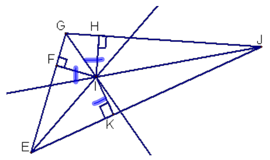


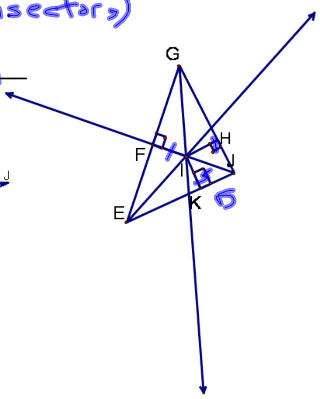




outside or inside of △

The incenter. (3 ∠ Bisectors)

$\overline{IK} \cong \overline{IF} \cong \overline{IH}$

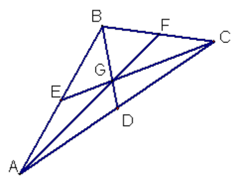


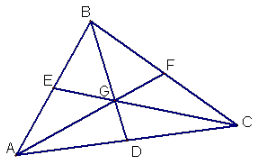




(inside of △)

The Centroid (3 medians) (inside of △)

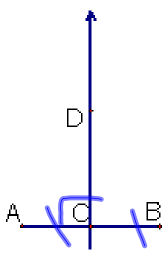
| | | |
|---|---|---|
| $BG = \frac{2}{3}BD$ | $CG = \frac{2}{3}CE$ | $AG = \frac{2}{3}AF$ |
| $GD = \frac{1}{3}BD$ | $GE = \frac{1}{3}CE$ | $GF = \frac{1}{3}AF$ |
| $GD = \frac{1}{2}BG$ | $GE = \frac{1}{2}CG$ | $GF = \frac{1}{2}AG$ |



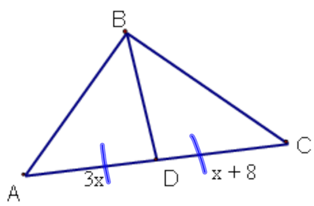




Examples:

1. $\overline{CD}$ is a ⊥ bisector of $\overline{AB}$.
   m∠DCA = 2x. Solve for x. 45
   $2x = 90$
   AC = 3y + 2, BC = 14. Solve for y. 4
   $3y + 2 = 14$
   $3y = 12$
   $y = 4$



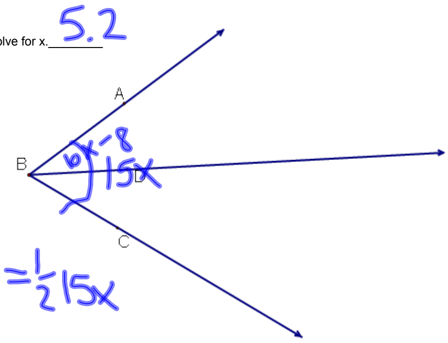

2. $\overline{BD}$ is a median in △ABC. Solve for x. 4




$3x = x + 8$
$x = 4$

3. $\overline{BD}$ bisects ∠ABC. Solve for x. 5.2

m∠ABC = 15x
m∠ABD = 10x - 8



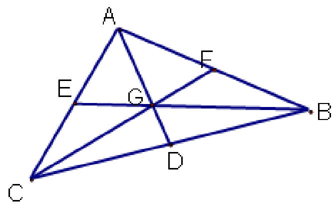

$10x - 8 = \frac{1}{2} 15x$

$x = 5.2$

4. G is the centroid.

AG = 7.4
AD = 6a
a = 1.85

GE = 5c
EB = 22.8
c = 1.52




$5c = \frac{1}{3} 22.8$

$c = 1.52$

$7.4 = \frac{2}{3} 6a$

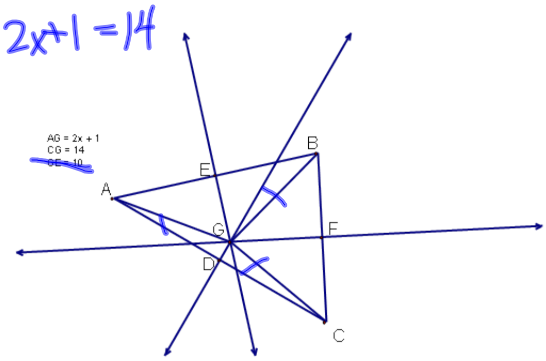
$7.4 = 4a$

$1.85 = a$

5. G is the circumcenter. x = 6.5.

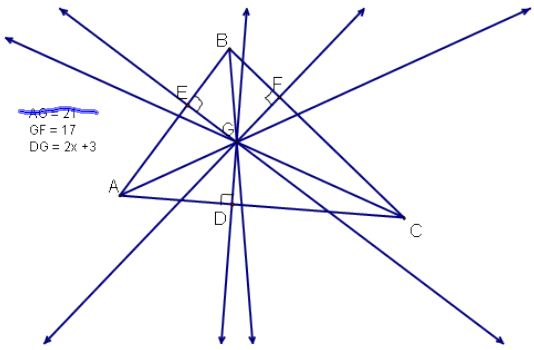
$2x + 1 = 14$

AG = 2x + 1
CG = 14
~~GE = 10~~




6. G is the incenter. x = 7.

~~AG = 21~~
GF = 17
DG = 2x +3




$2x + 3 = 17$

$x = 7$

HW
p243-244
6, 21, 22

Review
Info